Simple Optimizations for Applicative Array Programs for Graphics Processors

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GPUs are powerful, but difficult to program

- 1 TFLOP/s on modern GPUs; several times greater than CPUs

- Lots of code for simple operations:
  ```c
  float sum = 0;
  for (int i = 0; i < n; i += 1)
    sum += arr[i];
  ```
  in C takes ~150 lines of CUDA

- GPU code is data-parallel: you must decompose the problem’s data
Applicative array programming allows easy GPU use

- \( \text{vmap } f \, \text{xs} \)
  - Input: \( a \, b \, c \, d \)
  - Output: \( f(a) \, f(b) \, f(c) \, f(d) \)

- \( \text{vzipWith } f \, \text{xs } ys \)
  - Input: \( a \, b \, c \, d \)
  - Output: \( a' \, b' \, c' \, d' \)

- \( \text{vreduce } \oplus \, i \, \text{xs} \)
  - Input: \( a \, b \, c \, d \)
  - Output: \( i \oplus a \oplus b \oplus c \oplus d \)

- \( \text{vslice } (1, 2) \, \text{xs} \)
  - Input: \( a \, b \, c \, d \)
  - Output: \( b \, c \)
The Barracuda language supports these primitives on the GPU:

- Applicative: no side effects
- Compositional: primitives can be freely nested
- Deeply embedded within Haskell
- Functions on vectors, matrices, and scalars are the unit of compilation
Barracuda code resembles Haskell code on lists

\[
\text{rmse} :: \text{[Float]} \to \text{[Float]} \to \text{Float}
\]
\[
\text{rmse} \ x \ y = \sqrt{\text{sumDiff} / \text{fromIntegral} (\text{length} \ x)}
\]
\[
\text{where}
\]
\[
\text{sumDiff} = \text{sum} (\text{map} (^2) (\text{zipWith} (-) \ x \ y))
\]

\[
\text{rmse} :: \text{VExp Float} \to \text{VExp Float} \to \text{SExp Float}
\]
\[
\text{rmse} \ x \ y = \sqrt{\text{sumDiff} / \text{fromIntegral} (\text{vlength} \ x)}
\]
\[
\text{where}
\]
\[
\text{sumDiff} = \text{vsum} (\text{vmap} (^2) (\text{vzipWith} (-) \ x \ y))
\]
Barracuda code resembles Haskell code on lists

```haskell
rmse :: [Float] -> [Float] -> Float
rmse x y = sqrt (sumDiff / fromIntegral (length x))
  where
    sumDiff = sum (map (^2) (zipWith (-) x y))
```

Barracuda

```haskell
rmse :: VExp Float -> VExp Float -> SExp Float
rmse x y = sqrt (sumDiff / fromIntegral (vlength x))
  where
    sumDiff = vsum (vmap (^2) (vzipWith (-) x y))
```

Barracuda code works on GPU vectors, not lists
Barracuda code resembles Haskell code on lists

\[
\text{rmse} :: [\text{Float}] \rightarrow [\text{Float}] \rightarrow \text{Float} \\
\text{rmse} \ x \ y = \sqrt{\frac{\text{sumDiff}}{\text{fromIntegral} (\text{length} \ x)}} \\
\quad \text{where} \\
\quad \text{sumDiff} = \sum (\text{map} (\cdot^2) (\text{zipWith} (\cdot) \ x \ y))
\]

\[
\text{Barracuda functions are named differently}
\]

\[
\text{rmse} :: \text{VExp} \text{Float} \rightarrow \text{VExp} \text{Float} \rightarrow \text{SExp} \text{Float} \\
\text{rmse} \ x \ y = \sqrt{\frac{\text{sumDiff}}{\text{fromIntegral} (\text{vlength} \ x)}} \\
\quad \text{where} \\
\quad \text{sumDiff} = \text{vsum} (\text{vmap} (\cdot^2) (\text{vzipWith} (\cdot) \ x \ y))
\]
Barracuda functions construct abstract syntax trees

\[
\text{rmse} :: \text{VExp Float} \to \text{VExp Float} \to \text{SExp Float} \\
\text{rmse } x \ y = \sqrt{\frac{\text{sumDiff}}{\text{fromIntegral} (\text{vlength } x)}} \\
\text{where sumDiff} = \text{vsum} (\text{vmap } (^2) (\text{vzipWith } (-) x y))
\]

I.e., Barracuda is **deeply embedded** within Haskell
Barracuda ASTs are compiled into optimized CUDA code

The user writes these Barracuda ASTs

Barracuda compiler

Barracuda Functions

User’s C++ code

Barracuda runtime code

CUDA kernels

C++ wrapper functions

nvcc

GPGPU Application
Efficient GPU code exploits the memory hierarchy

NVIDIA Tesla C2050

- 48 KB Shared Memory
- 32 GPU Cores
- 3GB Device Memory

14 chips

Main Memory

100's

1000's
Nested array expressions are potentially troublesome

\[
\text{rmse} :: \text{VExp Float} \to \text{VExp Float} \to \text{SExp Float}
\]
\[
\text{rmse } x \ y = \sqrt{\frac{\text{sumDiff}}{\text{fromIntegral} (\text{vlength } x)}}
\]
\[
\text{where } \text{sumDiff} = \text{vsum} (\text{vmap} (^2) (\text{vzipWith} (-) x y))
\]

Naive compilation uses temporaries, multiple passes over data
CUDA computes on elements, not arrays

CUDA code is data-parallel: kernels
describe what happens at one location.

Array indexing laws allow for fusion:

\[
(vmap f xs)!i = f (xs!i)
\]

\[
(vzipWith f xs ys)!i = f (xs!i) (ys!i)
\]

\[
(vslice (b, e) xs)!i = xs!(e - b + i)
\]
Barracuda always applies the array indexing laws

Array fusion comes naturally during codegen, e.g.:

\[
(vmap f (vmap g xs))!i \rightarrow f (g (xs!i))
\]

\[
vmap f (vzipWith g xs ys)!i \rightarrow f (g (xs!i) (ys!i))
\]

\[
(vslice (b, e) (vmap f xs))!i \rightarrow f (xs!(e - b + i))
\]

\[
(vmap f (vslice (b, e) xs))!i \rightarrow f (xs!(e - b + i))
\]

\[
(vslice (b, e) (vslice (b’, e’) xs))!i \rightarrow xs!(e - b + e’ - b’ + i)
\]
Efficient GPU code exploits the memory hierarchy
Stencil operations involve redundant reads

A data-parallel CUDA *kernel* is run by many *threads* on the 14 GPU chips.

Stencil operations involve array elements in a neighborhood, resulting in several threads reading the same elements.
Barracuda automatically uses shared memory when useful

When multiple array subexpressions overlap, there is read redundancy, e.g.:

$$\text{as} = \text{vzipWith} \ (-) \ \text{zs} \ \text{ys}$$
$$\text{ys} = \text{vslice} \ (0, \ 6) \ \text{xs}$$
$$\text{zs} = \text{vslice} \ (1, \ 7) \ \text{xs}$$

<table>
<thead>
<tr>
<th>xs</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
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Elements b–g are read twice in the computation of \text{as}
Use of shared memory is only useful when

- array elements are read at least two times;
- it is known at compile-time that elements are read multiple times; and
- there are enough elements to amortize the added indexing costs.
Shared memory optimization examples

\[ as = \text{vzipWith} \ (-) \ zs \ ys \]
\[ ys = \text{slice} \ (0, 1022) \ xs \]
\[ zs = \text{slice} \ (1, 1023) \ xs \]

There are enough elements

\[ as = \text{vzipWith} \ (-) \ zs \ ys \]
\[ ys = \text{slice} \ (0, 511) \ xs \]
\[ zs = \text{slice} \ (512, 1023) \ xs \]

No elements are read multiple times

\[ as = \text{vzipWith} \ (-) \ zs \ ys \]
\[ ys = \text{slice} \ (0, 511) \ xs \]

No elements are read multiple times

\[ as = \text{vzipWith} \ (-) \ zs \ ys \]
\[ ys = \text{slice} \ (0, 1022) \ xs \]
\[ zs = \text{slice} \ (1, \text{vlength} \ xs) \ xs \]

Slices use only constant and vector length expressions; there are enough elements
A mix of existing and new benchmarks was used

- BLAS operations, Black-Scholes seen in Lee et al. (2009) and Mainland and Morrisett (2010)

- Weighted moving average, RMSE, forward difference used to show impact of optimizations

- Test system: 512MB NVIDIA GeForce 8800GT, CUDA 3.2
Barracuda performance is good

Runtime relative to hand-coded solutions

Faster

Slower

Number of array elements

SDOT
Black Scholes call options
SAXPY

Monday, February 14, 2011
Array fusion is essential for good performance

RMSE average kernel runtime

Fused version 1.1x faster

Fused version 1.7–2.9x faster

Manually unfused

With fusion
Use of shared memory greatly improves performance
Speedups are enabled by careful use of declarative programming

Barracuda gets speedups through better use of GPU memory.

Computation is moved into fast memory through array fusion and shared memory optimization.

These optimizations are easy to implement because the source language is applicative and has few primitives.